The Effects of Disorder on Elastic Modulus

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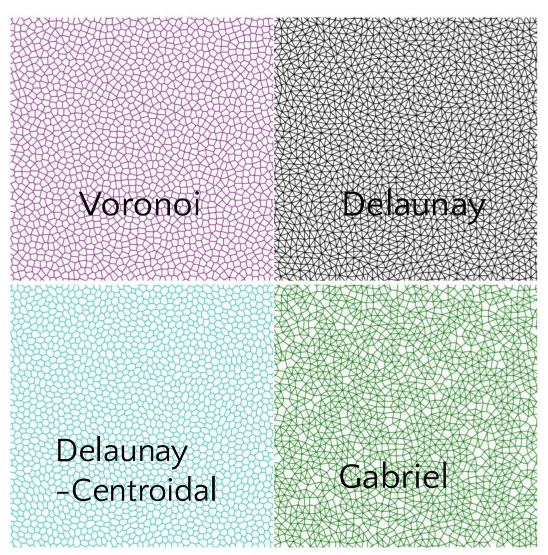
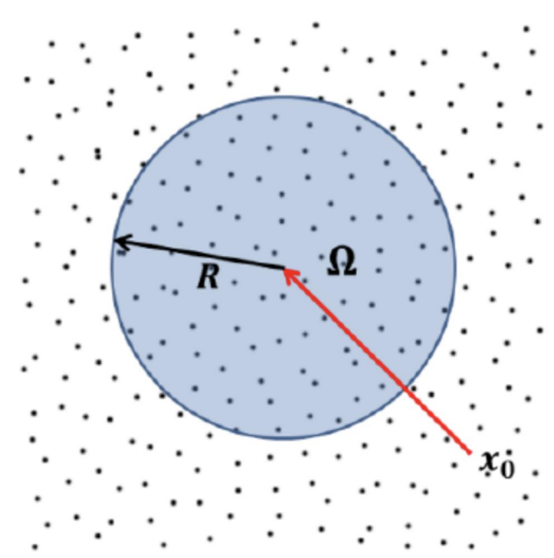
CMMI-2323341

Background

- **Hyperuniformity:** Point clouds that have density fluctuations suppressed at a long-length scale.
- Each point cloud starts from a uniform configuration of points separated by distance *d*. New point clouds are generated by giving the points a random kick of size *ad*.
- We work with four different sets of rules for connecting these disordered lattices.



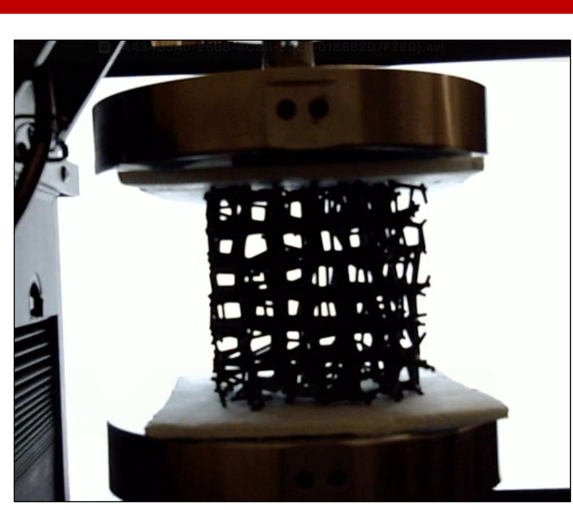
Adapted from Maher & Newhall "Characterizing the Hyperuniformity of Disordered Network Metamaterials" *PRE* 2025



Hyperuniformity is evaluated by measuring density fluctuations as a function of observation window size. Adapted from Torquato *Physics Reports* 2018.

- We generated 3D-printable files (STLs) of point clouds connected by the Voronoi tessellation (since that preserves hyperuniformity best) at varying degrees of disorder.
- STL Generation GitHub:

Experiment Set-Up

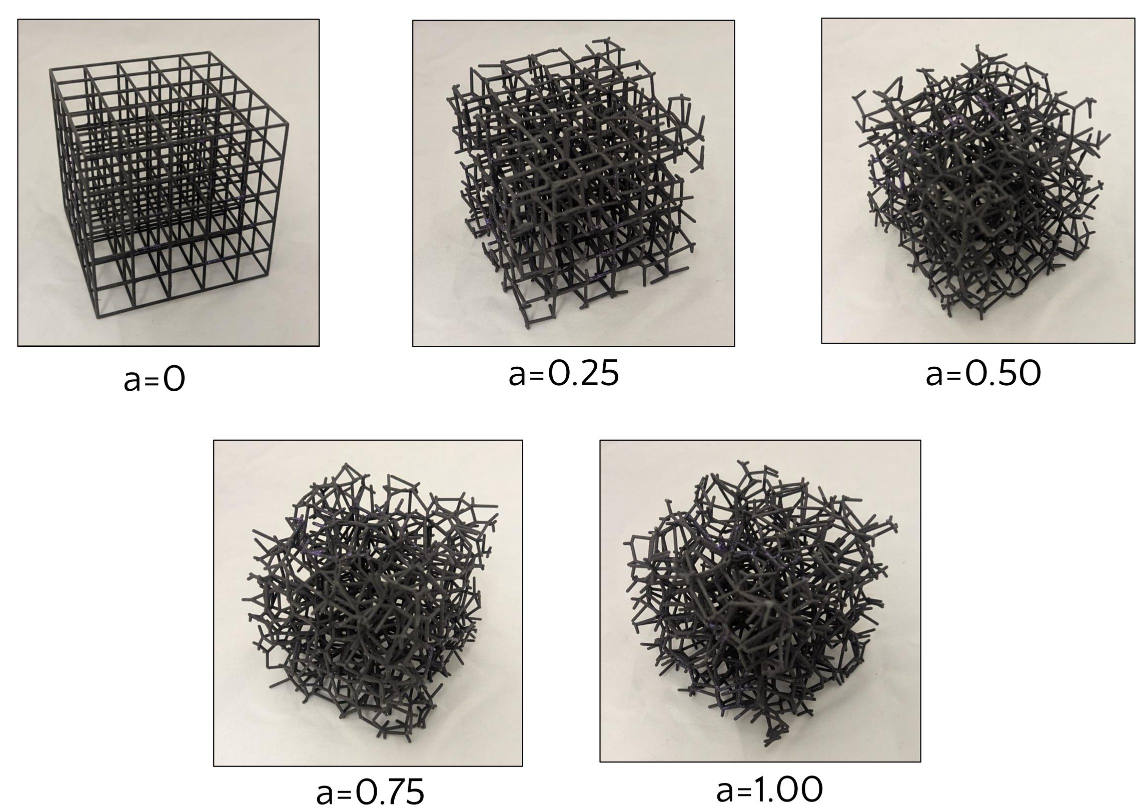


To determine the elastic modulus, we compress the samples on the Instron machine in EaRL. The plates above and below the sample move up/down for compression/decompression, under strain-controlled boundary conditions.

Question

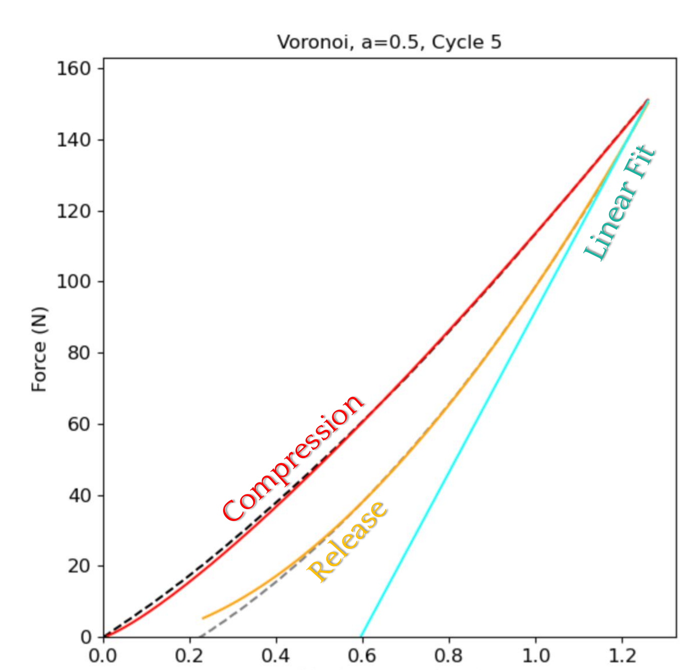
How does disorder affect the elastic modulus of a 3D network?

Algorithmic Disorder



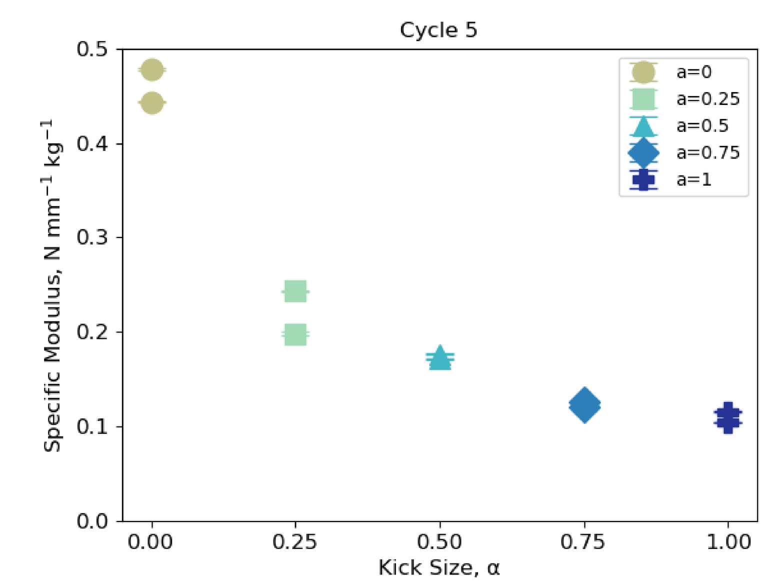
Progression of disorder for samples connected by the Voronoi tessellation.

Measurement of Elastic Modulus



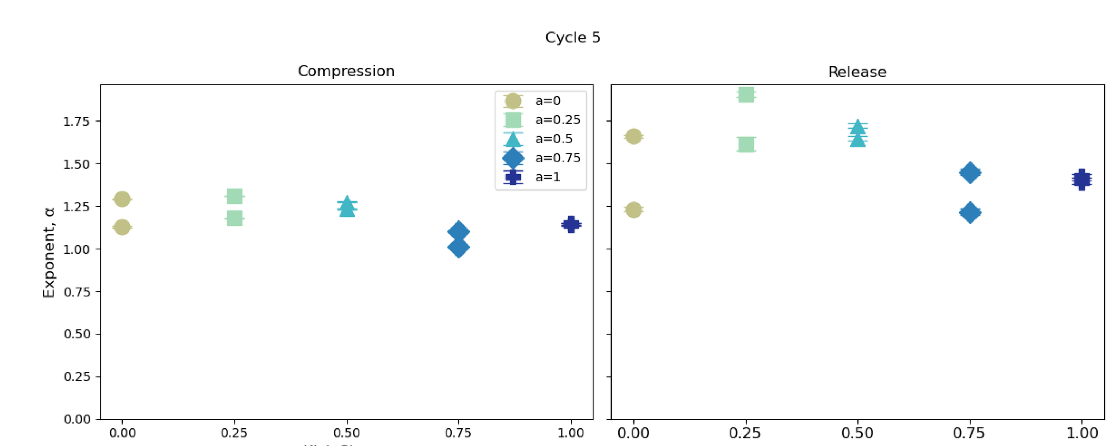
- Compression and Release are fit with $F = Ax^{\alpha}$
- The linear fit is measured from the Force/Displacement curve and converted to **elastic modulus** using the sample's dimensions.
- We fit the linear fit to the point of release because the system behaves approximately linearly here.
- To compare stiffness across samples of slightly varying mass, we divide out the effect of mass to obtain a **specific modulus (Pa/kg).**

Specific Modulus vs. Disorder



- **Disorder affects specific modulus.** What explains this?
- We are currently studying network measures which might explain this effect.

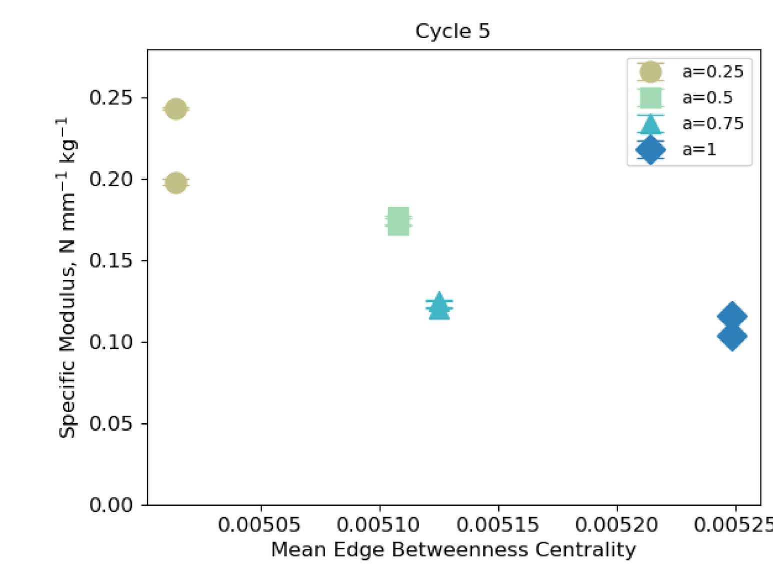
Exponents vs. Disorder



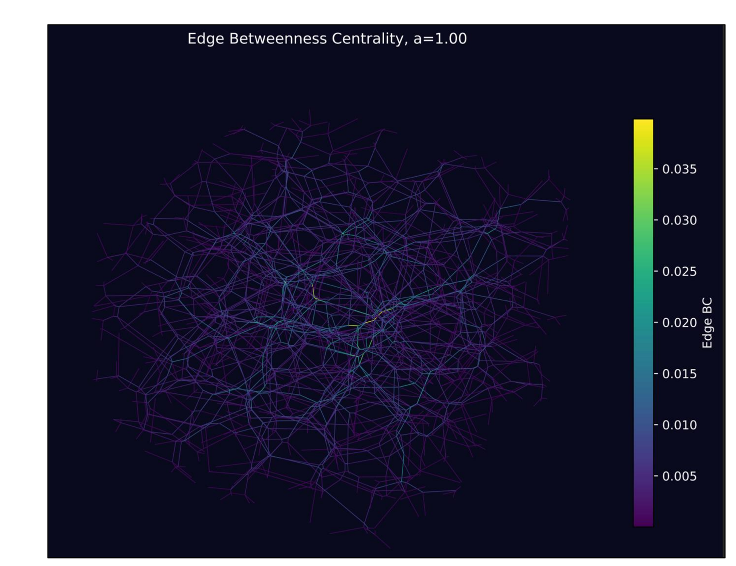
- Exponent refers to α in $F = Ax^{\alpha}$
- The release exponent is systematically higher than the compression exponent.
- Both exponents are approximately independent of the kick size, *a*.

Edge Betweenness Centrality

- **Edge Betweenness centrality** is a network measure of how often an edge lies on the shortest path between pairs of nodes.



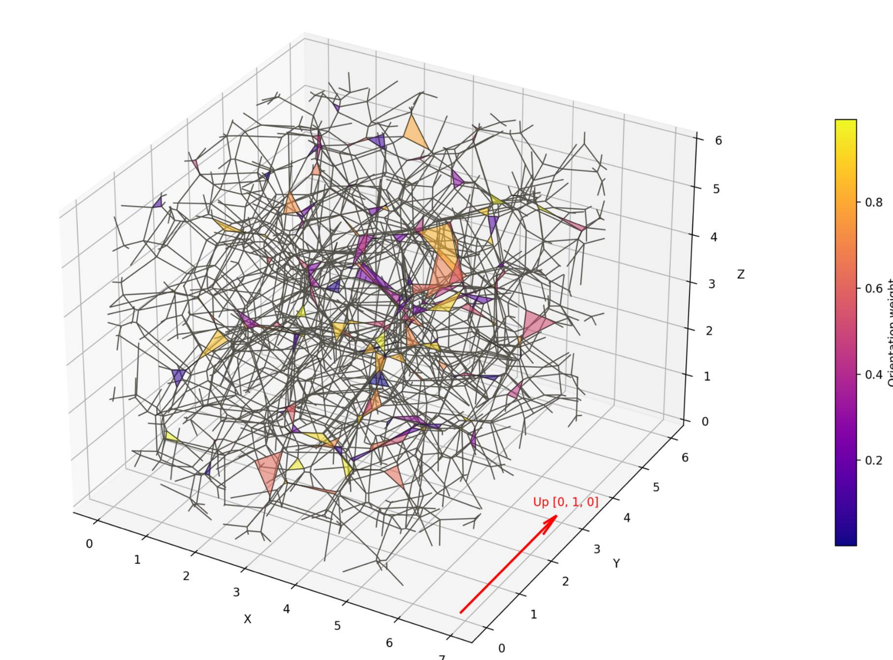
- Average betweenness centrality decreases as a function of kick size
- Below is an example of how a small number of high centrality edges may influence this result.



Visualization of Edge Betweenness Centrality

Work in Progress to Account for Compression Axis

- **Triangle Count:** Standard network science techniques simply count the number of triangles. Because the compression is along a single axis, we plan to instead weight each triangle by the dot product of the triangle normal vector and the direction of compression. Triangles oriented with their normal vector parallel to compression may have more impact on stiffness.
- **Betweenness Centrality Modification:** Only calculating betweenness centrality between the nodes on the top and bottom of the side being compressed.



In this image of the Voronoi a=1.00, the triangles are being weighted by their dot product. How much they are weighted is shown by the legend on the right.